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ON PROPAGATION OF LINEAR WAVES IN A CYLINDRICAL CHANNEL WITH A PERMEABLE WALL

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Problems on propagation of harmonic waves and of waves of finite duration in a cylindrical channel filled by a liquid or a gas and surrounded by a porous permeable space are considered. A wave equation describing the dynamics of small disturbances in a cylindrical channel with a permeable wall is obtained. Results of the analysis of wave evolution are presented for the cases of channel filling by a liquid and a gas.

The study of wave effects in tubes was begun in [1, 2] and developed in theoretical and experimental investigations [3-5]. Various aspects of filtration theory are presented and described in detail [6, 7]. However, the processes of wave propagation in channels surrounded by porous permeable space have received little attention in scientific literature, despite the importance of the problem, due to the widespread use of these models in science, engineering, and nature.

The present paper studies the evolution of harmonic waves and waves of finite duration in a cylindrical channel which is filled by a liquid or a gas, possesses a permeable wall, and is surrounded by porous space. The engineering and scientific aspects of the problem can, in particular, be associated with the solution of practical problems of exploration, drilling, and exploitation of boreholes. Results and conclusions are applicable as differentiated information about near-borehole processes, a theoretical basis for predicting and current estimates of the state of near-borehole zones, and for substantiation of the complex of techniques and technologies used to increase oil and gas output of beds and the productivity of boreholes.

1. We consider the propagation of small disturbances in a cylindrical channel in the presence of filtration processes through its permeable wall under the following assumptions: the channel and the incompressible skeleton of the porous space surrounding it are filled by the same medium (a liquid or a gas); the liquid is barotropic and its viscosity manifests itself only during filtration. Moreover, we assume that the wavelength is larger than the channel diameter.

Within the framework of the scheme of one-dimensional motion, medium flow in the channel and its filtration through the permeable wall to surrounding porous space can be described by the following system of linearized equations

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial w}{\partial z} = -\frac{2\rho_0 u}{r_0}, \quad r < r_0 ; \tag{1}$$

$$\rho_0 \frac{\partial w}{\partial t} + \frac{\partial P}{\partial z} = 0 , \quad r < r_0 ; \tag{2}$$

$$\frac{\partial P^{(1)}}{\partial t} = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P^{(1)}}{\partial r} \right), \quad \kappa = \frac{k_{\text{med}} \rho_0 C^2}{\mu m}, \quad r \geq r_{\text{med}} ; \tag{3}$$

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$$u^{(1)} = -\frac{k_{\text{med}}}{\mu}\frac{\partial P^{(1)}}{\partial r}, \quad r > r_{\text{med}}; \tag{4}$$

$$u^{(1)} = u\,(r_0/r_{\text{med}}), \quad P^{(1)} = P^{(1)}_{\text{med}}, \quad r = r_{\text{med}}. \tag{5}$$

We assume in general that there is a low-permeability crust on the outer surface of the channel wall; the rate of medium absorption through the crust is assigned as follows

$$u = h\,(P - P^{(1)}_{\text{med}}). \tag{6}$$

When the channel is surrounded by a porous space of finite length (filtration processes in propagation of disturbances take place in layers whose characteristic length is much smaller than the porous-wall thickness), we add one more boundary condition to the system of conditions (5), (6):

$$P^{(1)} = 0, \quad r \to \infty. \tag{7}$$

If the thickness of the permeable crust is small ($r_{\text{med}} - r_0 << r_0$) and hydraulic resistance during medium filtration through is negligible, then we write instead of (5) and (6)

$$u^{(1)} = u, \quad P^{(1)} = P, \quad r = r_0. \tag{8}$$

We have from Eqs. (1) and (2) with allowance for the equation of state

$$\frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} - \frac{\partial^2 P}{\partial z^2} = -\frac{2\rho_0}{r_0}\frac{\partial u}{\partial t}. \tag{9}$$

For the problem under consideration we can obtain, using relation (9) and accounting for the equation of piezoconductivity and boundary conditions (7), (8), the wave equation describing the dynamics of small disturbances in a cylindrical channel with a porous permeable wall.

To find the solution of Eq. (3) under conditions (7), (8) we use a Weber integral transform with kernel $rK(r, \lambda)$ [8], where

$$K(r, \lambda) = J_0(r_0\lambda)\,N_0(r\lambda) - J_0(r\lambda)\,N_0(r_0\lambda), \quad r_0 \le r < \infty.$$

Here $K(x)$ is McDonald function, which is determined by a first-order Hankel function, the imaginary part of which $N_0(x)$ is a zero-order Neuman function; $J_0(x)$ is a zero-order Bessel function. Multiplying Eq. (3) by the kernel $rK(r, \lambda)$ and integrating the obtained expression from r_0 to infinity, we have

$$\frac{\partial \hat{P}^{(1)}}{\partial t} = \kappa \int_{r_0}^{\infty} \frac{\partial}{\partial r}\left(r\frac{\partial P^{(1)}}{\partial r}\right) K(r, \lambda)\,dr,$$

$$P^{(1)}(z, \lambda, t) = \int_{r_0}^{\infty} P^{(1)}(z, r, t)\,rK(r, \lambda)\,dr.$$

Hence, with allowance for the conditions (7) and (8) we obtain the expression

$$\frac{\partial \hat{P}^{(1)}}{\partial t} = \kappa\left(\frac{2}{\pi}P - \lambda^2\hat{P}^{(1)}\right),$$

whose solution, satisfying the condition $\hat{P}^{(1)} = 0$, $t = 0$, is

$$\hat{P}^{(1)} = \frac{2\kappa}{\pi} \int_0^t P(z, \tau) \exp[\kappa\lambda^2 (\tau - t)] \, d\tau .$$

Using the Weber transform

$$\hat{P}^{(1)} = \int_{r_0}^{\infty} \frac{\hat{P}^{(1)}(z, \lambda, t) \lambda K(r\lambda)}{J_0^{(2)}(r_0\lambda) + N_0^2(r_0\lambda)} \, d\lambda ,$$

with allowance for

$$J_0(r_0\lambda) N_0'(r_0\lambda) - N_0(r_0\lambda) J_0'(r_0\lambda) = \frac{2}{\pi r_0 \lambda} ,$$

we have from (4)

$$u = -\frac{4}{\pi^2} \frac{k\kappa}{\mu r_0} \int_{r_0}^{\infty} \int_0^t \frac{P(z, \tau) \exp[-\kappa\lambda^2 (t - \tau)] \lambda}{J_0^2(r_0\lambda) + N_0^2(r_0\lambda)} \, d\lambda . \tag{10}$$

Substituting (10) into (6), we obtain the wave equation

$$\frac{1}{C^2} \left(\frac{\partial^2 P}{\partial t^2} - \frac{\partial}{\partial t} \left(\frac{8}{\pi^2} \right) \frac{m\kappa^2}{r_0^2} \int_0^t \int_{r_0}^{\infty} \frac{P(z, \tau) \exp[-\kappa\lambda^2 (t - \tau)] \lambda d\lambda d\tau}{J_0^2(r_0\lambda) + N_0^2(r_0\lambda)} \right) - \frac{\partial^2 P}{\partial z^2} = 0 , \tag{11}$$

which describes the evolution of small disturbances in a cylindrical channel with a porous permeable wall.

We seek the solution of the problem in the form of an attenuating traveling wave assuming that the wave propagates parallel to the coordinate axis toward its positive direction

$$P = A_p \exp[i(Kz - \omega t)] , \quad u = A_u \exp[i(Kz - \omega t)] ,$$
$$P^{(1)} = A_p^{(1)}(r) \exp[i(Kz - \omega t)] , \quad u^{(1)} = A_u^{(1)}(r) \exp[i(Kz - \omega t)] , \tag{12}$$

$$K = k + i\delta , \quad C_p = \omega / k .$$

From the condition of the existence of a nontrivial solution of this form we obtain the dispersion expression

$$K^2 = \omega^2 \left(1 - 2m \frac{R_{\text{med}} K_0'(yR_{\text{med}})}{y(K_0(yR_{\text{med}}) - \beta_i K_0'(yR_{\text{med}}))} \right) , \tag{13}$$

$$y = \sqrt{\left(-i \frac{\omega r_0^2}{\kappa} \right)} , \quad \beta_i = \frac{k_{\text{med}}}{r_0 \mu h} , \quad R_{\text{med}} = \frac{r_{\text{med}}}{r_0} .$$

where the parameter

$$|y| = \sqrt{\left(\frac{\omega r_0^2}{\kappa} \right)} = \frac{r_0}{r_\omega} , \quad r_\omega = \sqrt{\left(\frac{\kappa}{\omega} \right)}$$

has the physical meaning [9] of the ratio of the channel radius (r_0) to the characteristic depth of filtration wave penetration (r_ω) to the surrounding porous space, which expresses the distance at which the amplitude of filtration waves decreases by about twofold.

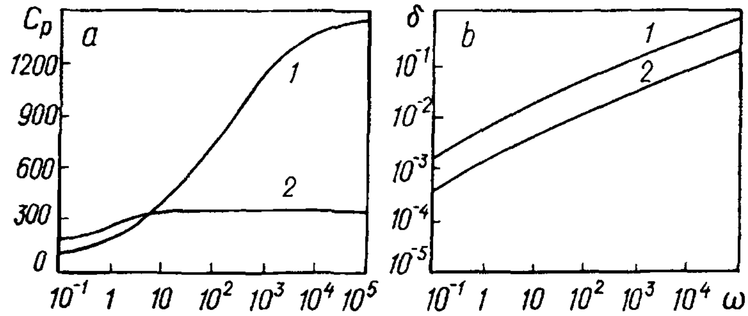


Fig. 1. Phase velocity (a) and coefficient of attenuation (b) in channel ($r_0 = 5\cdot10^{-2}$ m, $k_{med} = 10^{-12}$ m^2, $m = 0.2$) filled by water (1) ($\nu = 1.06\cdot10^{-6}$ m^2/sec, $C = 1.425\cdot10^3$ m/sec), air (2) ($\nu = 1.5\cdot10^{-5}$ m^2/sec, $C = 341$ m/sec). C_p, m/sec; δ, m^{-1}; ω, sec^{-1}.

If we ignore the hydraulic resistance of the channel wall during filtration ($h \to \infty$, $\beta_i \to 0$, $R_{med} \simeq 1$), we obtain from (13)

$$K^2 = \omega^2 (1 - 2m (\ln K_0 (y))' y^{-1}) C^{-2}. \tag{14}$$

Here $K_0(y)$ is a zero-order McDonald function for which the integral representation

$$K_0 (y) = \int_0^\infty \exp (- y \operatorname{ch} \xi)\, d\xi$$

holds.

Analysis of dispersion expression (14) within the range of high frequencies ($r_\omega << r_0$), for which the condition

$$|y| >> 1 \quad \text{or} \quad \omega >> \omega_\kappa \tag{15}$$

is satisfied (where $\omega_\kappa = \kappa/r_0^2$ is the characteristic frequency at which the depth of penetration of filtration waves is on the order of the channel radius [9]), gives asymptotic relations

$$C_p = C\,, \quad \delta = \frac{m}{r_0 C} \sqrt{\left(\frac{\kappa\omega}{2}\right)}\,. \tag{16}$$

Within the range of low frequencies ($r_\omega >> r_0$), which satisfy, besides the condition

$$|y| << 1 \quad \text{or} \quad \omega << \omega_\kappa\,, \tag{17}$$

the additional condition

$$|y|^2 \ln |y|| >> 2m\,,$$

the asymptotic relation for phase velocity and the coefficient of attenuation have the form

$$C_p = C\,, \quad \delta = - \frac{m\kappa}{r_0^2 C \,|\ln |y||}\,. \tag{18}$$

For the case of "extremely low" frequencies, when, besides condition (17), the condition

$$|y|^2 \,|\ln |y|| << 2m\,,$$

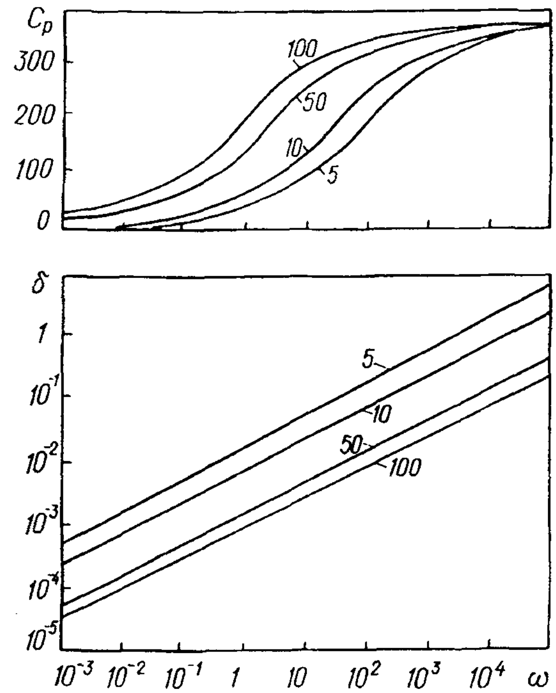


Fig. 2. Dispersion curves illustrating the effect of channel radius.

is also satisfied, we have the following relations:

$$C_p = C\frac{|y|\sqrt{|\ln|y||}}{\sqrt{m}}, \quad \delta = \frac{\omega\sqrt{m}}{C\,|y|\sqrt{|\ln|y||}}. \tag{19}$$

Analysis of expressions (16), (18), (19) shows that the velocity of propagation of harmonic disturbances in cylindrical channels surrounded by a porous permeable medium changes from zero ($C_p \ll C$) in the low-frequency range to a value close to the sound velocity in the medium ($C_p \simeq C$) in the region of high frequencies. The quantity C/ν is a physical parameter determining the dynamics of low-frequency disturbances. The rate of attenuation increases with a decrease in the compressibility and kinematic viscosity of the medium, which is determined by the sound velocity in the medium. The coefficient of attenuation at both low and high frequencies is inversely proportional to the channel radius and directly proportional to the porosity and permeability of the channel walls. For low-frequency disturbances this dependence is stronger.

Figure 1 presents dependences of the propagation velocity and the coefficient of attenuation on frequency in channels filled by water and air that were calculated by dispersion equation (14). As is seen from the curves, in a less viscous medium – water – the wave attenuation is stronger. For frequencies corresponding to the upper applicability limit of the studied model, which is determined by virtue of the above taken assumption ($\lambda = 2\pi C/\omega \gg 2r_0$, λ is the wavelength) from the condition $\omega \ll \omega^* = \omega C/r_0$ (in particular, at $r_0 = 10^{-2}$ m for a porous water-saturated space the frequency limiting the applicability range of the theoretical model $\omega_* = 3\cdot 10^5$ sec^{-1}), the difference in the coefficients of attenuation of disturbances in water and air is about 0.4 in the case illustrated.

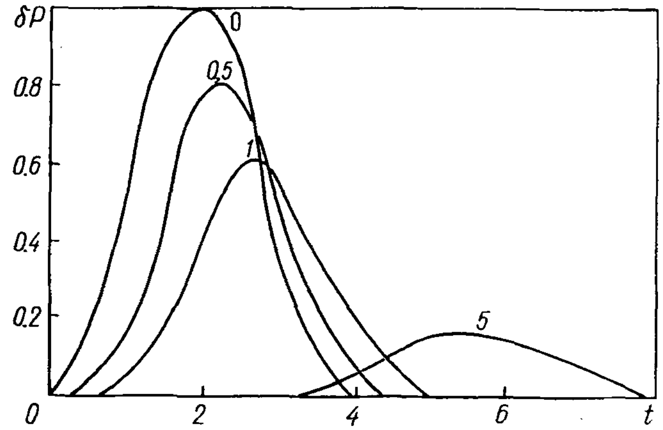
Figure 2 shows dispersion curves illustrating the effect of the channel radius. In the considered case the channel is filled by air. The figures at the curves correspond to the channel radius.

2. We consider the problem of the propagation of waves of finite duration in a cylindrical channel with permeable walls.

Let at the boundary of a semi-infinite region the pressure disturbance be specified as a function of time

$$P(0, t) = P^{(0)}(t), \tag{20}$$

with the function $P^0(t)$ satisfying the condition



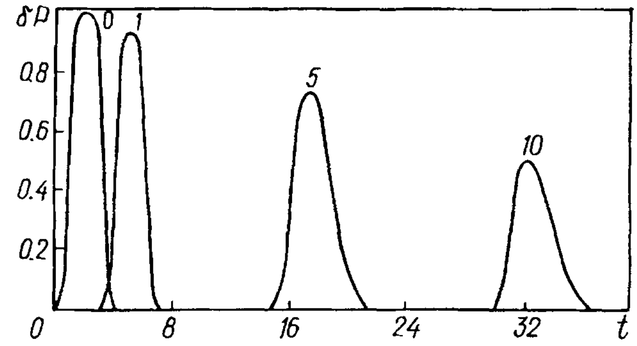


Fig. 3. Curve of pressure in channel ($r_0 = 5 \cdot 10^{-2}$ m, $k_{\text{med}} = 10^{-12}$ m^2, m = 0.2) with water. t, msec.

Fig. 4. Evolution of pressure pulse in channel ($r_0 = 5 \cdot 10^{-2}$ m, $k_{\text{med}} = \cdot 10^{-12}$ m^2, $m = 0.2$) filled by air.

$$|P^{(0)}| \to 0\,, \quad t \to \pm\infty\,.$$

The evolution of pulse disturbances is studied on the basis of the wave equation according to Fourier analysis.

For simplicity, we restrict ourselves to the consideration of long-term disturbances, during which the depth of filtration wave penetration into the porous space is much smaller than the channel radius.

For the considered frequency range we represent the wave equation, obtained similarly to (11), as

$$\frac{1}{C^2}\frac{\partial^2}{\partial t^2}\left(P + \frac{2m\sqrt{\kappa}}{\pi r_0}\int_{-\infty}^{t}\frac{P(z,\tau)}{\sqrt{t-\tau}}\,d\tau\right) - \frac{\partial P^2}{\partial z^2} = 0\,. \tag{21}$$

We sought the solution of Eq. (21) in the form

$$P(z, t) = \int_0^\infty \hat{P}(\omega)\exp\left(-i\left(K(\omega)\,z - \omega t\right)\right)d\omega\,.$$

On the basis of boundary condition (20) we can write for the function $\hat{P}(\omega)$

$$\hat{P}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P^{(0)}(t)\exp(i\omega t)\,dt\,,$$

As the dependence $K(\omega)$ we restrict ourselves to only its high-frequency branch, which is valid for $\omega >> \omega_\kappa$. Then from dispersion equation (14) we obtain the expression

$$K^2 = \omega^2\left(1 + \frac{2m}{y}\right)C^{-2}\,.$$

The evolution of weak impulse disturbance is numerically analyzed by algorithms of the fast Fourier transform [10-12]. Figure 3 illustrates the propagation and attenuation of a bell-shaped pulse disturbance of single amplitude with a length of $4 \cdot 10^{-3}$ sec in a channel filled by water, and Fig. 4, in a channel filled by air. The numbers at the curves correspond to the distance (in meters) from the site of signal initiation. It is seen from a comparative analysis of the curves that during pulse propagation both the attenuation of pulse amplitude and smearing of the signal shape take place. In this case the attenuation process in water is more intense than in air. Under the considered conditions in air the decrease in the wave amplitude is by about twofold at a distance of 10

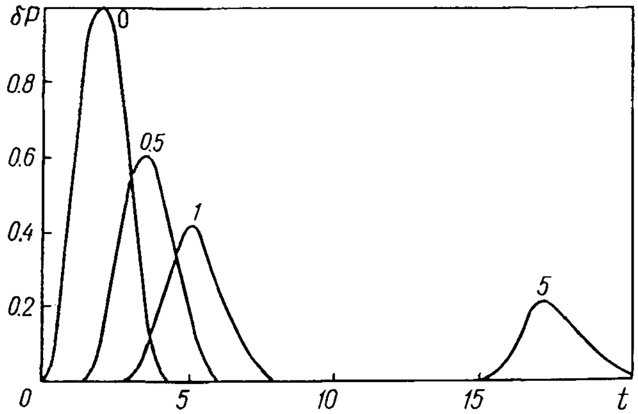


Fig. 5. Propagation and attenuation of pressure pulse in air-filled channel, $r = 5 \cdot 10^{-3}$ m.

m from the site of signal initiation, whereas in water at a distance of 1 m from the initiation site the wave amplitude is about half of its initial value. Figure 5 illustrates the evolution of a similar pulse in a channel filled by air the radius of which is 10 times smaller. A comparison of the graphs of Figs. 4 and 5 shows that the process of pulse disturbance attenuation is a strong function of the channel radius – the thinner the channel, the more intense the attenuation.

NOTATION

z, coordinate along the channel axis; P and ρ, disturbances of pressure and density, respectively; w, velocity of medium in channel in cross-section with coordinate z at time instant t; u, rate of filtration through channel wall; C, sound velocity in medium; μ, medium dynamic viscosity; k_{med} and m, permeability and porosity of space surrounding channel; κ, coefficient of piezoconductivity; r_0, radius of inner surface of channel wall; r_{med}, coordinate of outer boundary of low-permeability crust; $P^{(1)}_{\text{med}}$, pressure disturbance at interface between channel-wall outer surface and porous medium; K, wave vector; δ, C_{p}, and ω, coefficient of attenuation, phase velocity, and angular frequency of disturbances. Super- and subscripts: 0, undisturbed state; (1), radial distribution of parameter in porous space around channel; med, value of parameter in porous medium around channel; p, values pertaining to motion.

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